

A Conceptual Introduction to 'Muon Physics'

It's rather surprising that everyone, everywhere, has always at hand a totally reliable and maintenance-free source of unstable elementary particles for experimental study. The particles most easily studied by this method are *muons*, and their availability at earth's surface is due to the interaction of 'cosmic rays' with the earth's upper atmosphere. This write-up will introduce you to the simplest techniques of detecting these muons, and a method for acquiring the exponential decay curve for a sample of muons.

So, what are 'cosmic rays'? They are mostly protons, accelerated to high energies by mechanisms still unknown, somewhere in the Galaxy. A fraction of this resident population of high-energy (many GeV) protons continually rains down onto earth from space. Their free flight is interrupted when the protons encounter the nuclei of atoms in earth's upper atmosphere, and such encounters induce nuclear reactions (for example, producing radioactive carbon-14 out of nitrogen-14 nuclei). But some of those reactions also yield unstable pions, which decay (in order 10^{-8} s) to muons and neutrinos. The latter are very hard to detect, but the muons are charged (charge $\pm 1e$ for μ^\pm) and they will therefore interact with matter electromagnetically. Happily the muons' mass (which is about $105 \text{ MeV}/c^2$, or about $207 m_e$) gives them a rather long range in the atmosphere, and many survive to reach sea level.

So there is, everywhere on earth's surface, a steady flux of muons, of typical energy 2 GeV, arriving at a convenient though low rate. And a fraction of these muons will interact, electromagnetically, with any matter they encounter. In TeachSpin's 'Muon Physics' apparatus, we have them interact with a cylinder of plastic scintillator. The typical high-energy muon passes right through the scintillator, but in doing so, it causes some ionization, and deposits about 50 MeV of its energy in the scintillator. And some fraction of that energy gets converted to photons of light (that's what scintillator material is good at), and some fraction of that light reaches a photomultiplier tube, which converts the brief flash of light to a detectable pulse of electrons. [Happily, the vastly more frequent events are due to background radiation of the earth. These ambient beta and gamma rays, represent events of initial energy $\approx 1 \text{ MeV}$ or less, and create much weaker light pulses, which are easily discriminated against.]

So the scintillator/photomultiplier assembly, when properly configured, produces several electronic events every second, almost all of which are due to muons passing through the scintillator. But the truly exciting fraction of those events is due to the muons which arrive with much-less-than-average kinetic energy, because such muons can lose enough energy in the scintillator to come to *rest* inside it. In coming to rest, they will deposit the last of their kinetic energy, typically of order 50 MeV, so they will still produce a scintillator flash as they come to rest.

Those muons then live a relatively long time, on the order of order *microseconds*, inside the scintillator; but eventually each decays, typically via the reaction $\mu^\pm \rightarrow e^\pm + 2 \nu$. Nearly all of the rest energy (105 MeV) of the stopped muons appears as kinetic energy of the three particles; on average, the e^\pm (positron or electron) gets a third of this energy, about 35 MeV.

(The two neutrinos, ν , carry away the rest, undetectably.) But such an e^\pm is a charged particle, itself certain to cause ionization as it moves through the scintillator. Happily the typical energy deposited in this ionization process is about the same as that deposited by a muon-in-transit, or a muon stopping, so the very same scintillator/PMT configuration is also suitable for detecting stopped muons subsequently decaying at rest.

But how can you tell the difference between these two sorts of events, if they deposit comparable energy? If muons pass through at (say) 5 events per second, then the mean interval between muon events is the reciprocal, 0.2 s or 200 ms of 'waiting time'. Of course there's a whole distribution of waiting times, since the muons arrive randomly, uncorrelated in time. But clearly there's a very small probability of muon events at the much smaller time separation of (say) $2 \mu\text{s} = 0.002 \text{ ms}$, since it's so tiny compared to the typical waiting time of 200 ms.

That's the clue to the strategy: use an 'electronic stopwatch' with full-scale range of $20 \mu\text{s}$, and start it running afresh at every event coming from the scintillator/PMT. You'll get about 5 such 'start' events per second, and very few will be followed by another event within just $20 \mu\text{s}$. So mostly the 'stopwatch' will reach its full range without a second event to 'stop' it. But the muons that *do* come to rest in the scintillator will provide the exceptions -- they will all decay in place, and a fraction of those decays will produce a second, successor, event emerging from the scintillator.

So the method is -- start that stopwatch for every event, and stop it at

- either a) $20 \mu\text{s}$ later, if there's no successor event,
- or b) at the successor event, if there is one.

If muons really do stop in, and decay within, the scintillator, there should be an excess of these b)-type events. In practice, the frequency of such a stop-followed-by-a-decay pair of scintillation events is about one per minute.

For any individual pair of scintillator flashes, say $2 \mu\text{s}$ apart in time as measured by the stopwatch, there's no way to tell if *that particular pair* was due to

- A) a first muon passing through, followed by a second muon passing through,
- or B) a first muon entering and stopping, followed by that muon decaying.

But if you wait to acquire a sample of (say) 100 such pair events, and *histogram* the time-intervals as measured by the stopwatch, you'll see that histogram has

- A) a flat-in-time distribution of type-A pairs of uncorrelated events,
- and overlying it, and dominating it at short times,
- B) a decaying-in-time distribution of type-B pairs of correlated events.

In fact, that decay-in-time distribution should be exponentially decaying in time, and should represent the 'survival curve' of muons. It won't be the survival time of muons since their creation (high in the atmosphere), but instead the survival time of muons since their arrival in the scintillator. But happily, for an exponential decay process, the result looks the same either way! So the 'Muon Physics' apparatus can generate, in an hour or two, a histogram which can be interpreted to give the half-life of muons in matter.

You can 'least-squares fit' the histogram to a function of the form

$$N(t) = B e^{-\left[\frac{t}{\tau}\right]} + A, \text{ which can also be written as } B \exp(-t / \tau) + A,$$

where τ gives the mean lifetime (that's $\ln 2$, or ≈ 0.693 , times the half-life), and you can compare the lifetime of muons in matter to the "book value" for muons in vacuum. A long enough run (perhaps of a whole week's duration) might give you enough statistical precision to see the difference between these numbers. They differ because negative muons in matter get captured, first into Bohr orbits around nuclei, then via nuclear reactions by the nuclei themselves, so there's an additional removal mechanism for muons in matter, compared to muons in vacuum. Happily, the difference is small for the low-Z materials of which the scintillator is made.

Long before you get that statistical power, you'll measure $\tau \approx 2 \mu\text{s}$, and you'll be up against a much bigger 'cognitive dissonance'. If muons are produced by pion decay in the upper atmosphere, they're born about 50 km above your head. If they have energies of about 1 GeV, then they're pretty relativistic, and move at $v \approx c$. So how long should they take to reach sea level?

$$\text{Answer: } t = d/v \approx (5 \times 10^4 \text{ m}) / (3 \times 10^8 \text{ m/s}) \approx 2 \times 10^{-4} \text{ s or } 200 \mu\text{s}.$$

And what fraction should survive a descent lasting that long?

$$\text{Answer: of order } e^{-\left[\frac{t}{\tau}\right]} \approx e^{-\left[\frac{200\mu\text{s}}{2\mu\text{s}}\right]} = e^{-100} \approx 10^{-44}.$$

That number is so close to zero that we'd basically see *no* muons reaching sea level, if it were not for the glamorous effects of special relativity!

There are two ways to think about this relativistic effect, depending on your choice of reference frame. Put your self in a frame at rest with respect to the earth, and the high-energy muons, of kinetic energy about 2 GeV, large compared to their rest energy of 0.1 GeV, must have Lorentz factor $\gamma = (1 - v^2/c^2)^{-1/2} \approx 20$. So in this frame, their lifetimes are *time-dilated*, by factor $\gamma \approx 20$, from about 2 μs to about 40 μs . And with that time-dilated lifetime, the fraction expected to survive a descent lasting 200 μs is respectably large, of order

$$e^{-\left[\frac{10\mu\text{s}}{2\mu\text{s}}\right]} = e^{-5} \approx 10^{-2}.$$

Sure enough, at earth's surface we see that non-negligible fraction that has survived.

But what if we live instead in the *muons'* rest frame? The principle of relativity says we're entitled to do so. Then there's *no* time-dilation, so why doesn't that put us back at survival fraction e^{-100} ? The reason is that in *this* rest frame, the whole earth is moving toward the stationary muons, at $v \approx c$, in fact with Lorentz factor $\gamma \approx 20$. Now that factor gives the *length_contraction* of the 50-km distance that the muons have to traverse, and decreases it to 2.5 km. In this frame of reference, then, the time taken to traverse the distance from production to detection is

$$t = d/v \approx (2.5 \times 10^3 \text{ m}) / (3 \times 10^8 \text{ m/s}) \approx 1 \times 10^{-5} \text{ s or } 10 \mu\text{s}.$$

The survival fraction of muons, computed in their own rest-frame, is given by

$$e^{-\frac{[10\mu\text{s}]}{[2\mu\text{s}]}} = e^{-5} \approx 10^{-2}$$

As we expected, the survival fraction is again $\approx 10^{-2}$.

So there's lots of physics to think about in this 'Muon Physics' apparatus; and all this still leaves out the electronics that can be studied. There are ways to generate actual pairs of light pulses, of your choice of time separation, and inject them right into the scintillator. There's the ability to choose the high-voltage bias applied to the photomultiplier tube, to change its gain. There's a chance to look at the actual high-speed electronic pulses emerging from the PMT. Of course there are any number of statistical tests that can be applied to the series of muon arrival events. If you have access to a road that runs up a mountain, you can actually measure the muon arrival rate as a function of altitude, and confirm that the muons are decaying (in flight) at only a time-dilated rate. Best of all, your muon source is as reliable as sunshine, and operates continuously for no payment, with no radiation-source paperwork to keep in order!